Polarization state of quadratic spatial optical solitons

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We derive governing equations that determine a full polarization state of transversely two-dimensional spatial solitons in a bulk anisotropic medium with the second-order nonlinearity. Based on nonlinear vectorial Maxwell's equations and approximation of slowly varying envelopes, our approach describes also lowest-order nonparaxial effects, however the most important factor governing radiation polarization is the medium anisotropy. This factor results in mixing of orthogonal components of electric field of quadratic soliton that consists of coupled beams at the fundamental frequency and its second harmonics. For the case of weak anisotropy we determine the soliton polarization state by a perturbation method; it turns out that it is elliptical and changing over the soliton transverse section. The approach allows generalization to the case of optical parametric oscillators.

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I. INTRODUCTION

Temporal and spatial solitons in media with quadratic optical nonlinearity predicted by Karamzin and Sukhorukov [1], are intensively studied now both experimentally and theoretically because of their high potential for applications to all-optical signal processing (see review [2] and references therein). More detailed theory and numerical simulations were given for the temporal quadratic solitons, including recent direct solution of full-wave vectorial nonlinear Maxwell's equations [3]. As for the spatial transversely two-dimensional (2D) quadratic solitons, their theory is not so developed and is based until now mainly on a phenomenological approach not deduced directly from the initial Maxwell's equations. The complexity of the theoretical problem is connected with effect of anisotropy essential for phase matching that results in more sophisticated form of radiation diffraction [4,5] and difference between directions of radiation wave vectors and energy flows (Poynting vectors).

The goal of the present paper is the consistent derivation of equations describing the full polarization state of stationary spatial 2D quasiparaxial optical solitons in bulk media with quadratic nonlinearity. We start with the full vectorial nonlinear Maxwell's equations and reduce them to the case of bichromatic radiation to coupled equations for electric fields at the fundamental and second harmonics (Sec. II). Then in Sec. III we apply the approach of slowly varying envelope to deduce the governing equations for electric field transverse components of sufficiently wide solitons. For the sake of simplicity we consider the case of uniaxial crystal with symmetry like for the kotassium ditiydrogen phosphate (KDP) crystal. These equations' analysis and approximate solutions are described in Sec. IV, and a final discussion is presented in Sec. V. Note that similar equations were derived recently for the case of weak anisotropy and biaxial crystal [6]. As far as we know, our results demonstrate for the first time that quadratic solitons have complicated polarization structure and are characterized by elliptical polarization changing over the soliton transverse section. This feature is important for a wide area of nonlinear optics and nonlinear physics dealing with high-power radiation propagation in anisotropic media.

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II. THE PROBLEM AND INITIAL EQUATIONS

Propagation of optical radiation in crystals is governed by Maxwell's equations that for a nonmagnetic and nonconducting medium without free charges take the form (in Gaussian units)

$$\nabla\times(\nabla\times\tilde{\tilde{\mathbf{E}}})+\frac{1}{c^2}\frac{\partial^2\tilde{\tilde{\mathbf{D}}}}{\partial t^2}=0,\quad \nabla\cdot\tilde{\tilde{\mathbf{D}}}=0. \tag{1}$$

Here $\tilde{\tilde{\mathbf{E}}}$ and $\tilde{\tilde{\mathbf{D}}}$ are the electric field and flux density vectors, respectively, c is the speed of light in vacuum, and t is time. Further we assume that the interaction between the fundamental wave (with frequency $\omega_1=\omega$) and its second harmonics (with frequency $\omega_1=2\omega$) is nearly phase matched, whereas all higher harmonics are far from being phase matched. Then

$$\tilde{\tilde{\mathbf{E}}}=\sum_{j=1}^{2}\tilde{\mathbf{E}}_j(\mathbf{r},t)e^{-i\omega_j t}+\text{c.c.},\quad \tilde{\tilde{\mathbf{D}}}=\sum_{j=1}^{2}\tilde{\mathbf{D}}_j(\mathbf{r},t)e^{-i\omega_j t}+\text{c.c.}, \tag{2}$$

and it follows from Eqs. (1)

$$\nabla\times(\nabla\times\tilde{\mathbf{E}})-\frac{\omega_j^2}{c^2}\tilde{\mathbf{D}}_j=0,\quad \nabla\cdot\tilde{\mathbf{D}}_j=0. \tag{3}$$

Let us decompose flux density vectors in the linear and nonlinear, with respect to electric field, parts

$$\tilde{\mathbf{D}}_j=\hat{\varepsilon}_j\tilde{\mathbf{E}}+4\pi\tilde{\mathbf{P}}_j\,. \tag{4}$$

Here $\hat{\varepsilon}_j$ are second-rank tensors of linear dielectric permittivity and $\tilde{\mathbf{P}}_j$ are the induced nonlinear electric polarizations.

It is convenient to give their form in the referential of the main crystalline axes (''crystallographic'' system of coordinates X, Y, Z, that correspond to lower indices 1,2,3). Then the linear dielectric permittivity tensor is diagonal and for uniaxial crystals takes the form

$$\hat{\varepsilon}_j=\begin{pmatrix}\varepsilon_{\perp j} & 0 & 0\\ 0 & \varepsilon_{\parallel j} & 0\\ 0 & 0 & \varepsilon_{\parallel j}\end{pmatrix}. \tag{5}$$

In crystals without inversion center, the main contribution to the nonlinear electric polarization is quadratic in the electric field:

$$\tilde{\mathbf{P}}_{1m}=\sum_{n,l=1}^{3}\chi^{(1)}_{mnl}\tilde{\mathbf{E}}_{2n}\tilde{\mathbf{E}}^{*}_{1l},\quad \tilde{\mathbf{P}}_{2m}=\sum_{n,l=1}^{3}\chi^{(2)}_{mnl}\tilde{\mathbf{E}}_{1n}\tilde{\mathbf{E}}_{1l}. \tag{6}$$

Now one can rewrite Eq. (3) in the form

$$\nabla\times(\nabla\times\tilde{\mathbf{E}}_j)-\frac{\omega_j^2}{c^2}\hat{\varepsilon}_j\tilde{\mathbf{E}}_j-4\pi\frac{\omega_j^2}{c^2}\tilde{\mathbf{P}}_j=0,\quad j=1,2. \tag{7}$$

Spatial solitons we are interested in propagate along the longitudinal axis z and are characterized by constant transverse shapes of the electric fields, therefore

$$\tilde{\mathbf{E}}_j=\mathbf{E}_j(x,y)e^{i\Gamma_j z},\quad j=1,2. \tag{8}$$

Here x and y are the transverse coordinates, and the propagation constants $\Gamma_1=\Gamma$ and $\Gamma_2=2\Gamma$ were introduced for the fundamental and second harmonics, respectively. Relations between the ''crystallographic'' (X,Y,Z) and ''light'' (x,y,z) systems of coordinates are given by the Euler's angles θ (angle between the axes Z and z) and φ (angle between the axis X and projection of axis z on the plane (X,Y), see also [3]). More exactly, to get the ''light'' coordinate system, we first rotate the ''crystallographic'' system with the fixed axis Z at angle φ when the axis Y reaches the axis y, and next rotate the system with the fixed axis y at angle θ. Note that for a plane wave linear propagating along the axis z, we have, depending on polarization state,

$$E_y\neq 0,\quad n_o=\sqrt{\varepsilon_\perp} \tag{9}$$

for an ordinary wave, and

$$E_x,E_y\neq 0,\quad n_e^2=\frac{\varepsilon_\perp\varepsilon_\parallel}{\varepsilon_\perp\sin^2\theta+\varepsilon_\parallel\cos^2\theta} \tag{10}$$

for an extraordinary wave, where $n_{o,e}$ are corresponding refractive indices. Phase matching corresponds to the condition $n_o^{(j)}\approx n_e^{(3-j)}$ that is satisfied for the appropriate choice of angle θ.

Now we can exclude the field longitudinal dependence using the identity

$$\nabla\times(\nabla\times\tilde{\mathbf{E}}_j)=\nabla(\nabla\cdot\tilde{\mathbf{E}}_j)-(\boldsymbol{\Delta}_\perp-i\Gamma_j^2)\tilde{\mathbf{E}}_j,$$

where the transverse Laplacian was introduced

$$\boldsymbol{\Delta}_\perp=\nabla_\perp\cdot\nabla_\perp=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2},\quad \nabla_\perp=\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y}\right).$$

Let us introduce also 2D vectors consisting of transverse components of the electric field and polarization

$$\mathbf{E}_\perp=(E_x,E_y),\quad \mathbf{P}_\perp=(P_x,P_y).$$

For transverse components of vector Eq. (7) we have

$$\nabla_\perp^2\mathbf{E}_\perp+\frac{\omega_j^2}{c^2}(\hat{\varepsilon}\mathbf{E})_\perp-\Gamma^2\mathbf{E}_\perp-\nabla_\perp(\nabla_\perp\cdot\mathbf{E}_\perp+i\Gamma E_z)+4\pi\frac{\omega^2}{c^2}\mathbf{P}_\perp=0. \tag{11}$$

The longitudinal component of the electric field E_z can be expressed via the field transverse components from the second of Eqs. (1)

$$i\Gamma(\hat{\varepsilon}\mathbf{E})_z+\nabla_\perp\cdot(\hat{\varepsilon}\mathbf{E}_\perp)+4\pi i\Gamma P_z+4\pi\nabla_\perp\cdot\mathbf{P}_\perp=0. \tag{12}$$

In such a way it is possible to deduce a closed equation for transverse components of the soliton electric field, which is a goal of the next section.

III. GOVERNING EQUATION

Let us compare the order of magnitude of the four terms in the left-hand side of Eq. (12). To do this we allow that the soliton width w is larger as compared with the light wavelength λ $(w\gg\lambda)$ and, correspondingly, the propagation constant Γ is close to the linear wave number $k=2\pi/\lambda$. Then the second term is about $kE/(kw)$, the third $\sim kE/(kw)^2$ and the last term $\sim kE/(kw)^3$. Because $kw\gg 1$, we can neglect terms with nonlinear polarization and get purely linear equation

$$i\Gamma(\hat{\varepsilon}\mathbf{E})_z+\nabla_\perp\cdot(\hat{\varepsilon}\mathbf{E}_\perp)\approx 0. \tag{13}$$

In the ''crystallographic'' system of coordinates

$$\hat{\varepsilon}\mathbf{E}=\begin{pmatrix}\varepsilon_\perp & 0 & 0\\ 0 & \varepsilon_\parallel & 0\\ 0 & 0 & \varepsilon_\parallel\end{pmatrix}\begin{pmatrix}E_X\\ E_Y\\ E_Z\end{pmatrix}=\begin{pmatrix}\varepsilon_\perp E_X\\ \varepsilon_\parallel E_Y\\ \varepsilon_\parallel E_Z\end{pmatrix}.$$

Then in the ''light'' system of coordinates

$$\hat{\varepsilon}\mathbf{E}=\begin{pmatrix}E_x(\varepsilon_\perp\cos^2\theta+\varepsilon_\parallel\sin^2\theta)+E_z(\varepsilon_\perp-\varepsilon_\parallel)\sin\theta\cos\theta\\ E_y\varepsilon_\perp\\ E_x(\varepsilon_\perp-\varepsilon_\parallel)\sin\theta\cos\theta+E_z(\varepsilon_\perp\sin^2\theta+\varepsilon_\parallel\cos^2\theta)\end{pmatrix}.$$

Now for a plane wave we have

$$E_z=-\frac{1}{\varepsilon_\theta}(\varepsilon_\perp-\varepsilon_\parallel)\sin\theta\cos\theta E_x,$$

while for a wide soliton (the next iteration)

$$E_z=-\frac{1}{\varepsilon_\theta}(\varepsilon_\perp-\varepsilon_\parallel)\sin\theta\cos\theta E_x+\frac{i}{\Gamma\varepsilon_\theta}\left[\left(\varepsilon_\perp\cos^2\theta+\varepsilon_\parallel\sin^2\theta-\frac{1}{\varepsilon_\theta}(\varepsilon_\perp-\varepsilon_\parallel)^2\sin^2\theta\cos^2\theta\right)\frac{\partial E_x}{\partial x}+\varepsilon_\perp\frac{\partial E_y}{\partial y}\right], \tag{14}$$

where

$$\varepsilon_\theta=\varepsilon_\perp\sin^2\theta+\varepsilon_\parallel\cos^2\theta.$$

Substituting expression (14) for the longitudinal electric field component in Eq. (11), we find the governing equations

$$\nabla_\perp^2E_x^{(j)}+\left(\frac{\omega_j^2}{c^2}n_{ej}^2-\Gamma_j^2\right)E_x^{(j)}+i\beta_{xx}^{(j)}\frac{\partial E_x^{(j)}}{\partial x}+i\beta_{xy}^{(j)}\frac{\partial E_y^{(j)}}{\partial y}+\gamma_j\frac{\partial^2E_x^{(j)}}{\partial x^2}+\delta_j\frac{\partial^2E_y^{(j)}}{\partial x\partial y}+4\pi\frac{\omega_j^2}{c^2}P_x^{(j)}=0, \tag{15}$$

$$\nabla_\perp^2E_y^{(j)}+\left(\frac{\omega_j^2}{c^2}n_{oj}^2-\Gamma_j^2\right)E_y^{(j)}+i\beta_{xy}^{(j)}\frac{\partial E_x^{(j)}}{\partial y}+\gamma_j\frac{\partial^2E_x^{(j)}}{\partial x\partial y}+\delta_j\frac{\partial^2E_y^{(j)}}{\partial y^2}+4\pi\frac{\omega_j^2}{c^2}P_y^{(j)}=0. \tag{16}$$

Here

$$\beta_{xx}^{(j)}=\frac{\varepsilon_\perp^{(j)}-\varepsilon_\parallel^{(j)}}{\varepsilon_\perp^{(j)}\varepsilon_\parallel^{(j)}}\sin 2\theta\Gamma_j n_{ej}^2\left(1+\frac{n_{ej}^2}{2}\frac{\omega_j^2}{c^2\Gamma_j^2}\right),$$

$$\beta_{xy}^{(j)}=\frac{\varepsilon_\perp^{(j)}-\varepsilon_\parallel^{(j)}}{2\varepsilon_\perp^{(j)}\varepsilon_\parallel^{(j)}}\sin 2\theta\Gamma_j n_{ej}^2,$$

$$\gamma_j=\frac{\varepsilon_\perp^{(j)}-\varepsilon_\parallel^{(j)}}{\varepsilon_\perp^{(j)}\varepsilon_\parallel^{(j)}}n_{ej}^2\cos 2\theta-\left(\frac{\varepsilon_\perp^{(j)}-\varepsilon_\parallel^{(j)}}{2\varepsilon_\perp^{(j)}\varepsilon_\parallel^{(j)}}\right)^2n_{ej}^4\sin^2 2\theta,$$

$$\delta_j=\frac{\varepsilon_\perp^{(j)}-\varepsilon_\parallel^{(j)}}{\varepsilon_\perp^{(j)}\varepsilon_\parallel^{(j)}}n_{ej}^2\cos^2\theta.$$

Note that the nonlinear polarization components are taken in Eqs. (15), (16) in the ''light'' coordinates. They are linear combinations of the components in the ''crystallographic'' system of coordinates (6). To simplify the expressions, let us take the case of crystal with the $\bar{4}2m$ point group of symmetry, like the KDP crystal. Then there is only one independent value of nonlinear permittivity χ, and in the ''crystallographic'' system of coordinates

$$P_X^{(1)}=\chi[E_Y^{(2)}E_Z^{(1)*}+E_Z^{(2)}E_Y^{(1)*}],$$

$$P_Y^{(1)}=\chi[E_X^{(2)}E_Z^{(1)*}+E_Z^{(2)}E_X^{(1)*}],$$

$$P_Z^{(1)}=\chi[E_X^{(2)}E_Y^{(1)*}+E_Y^{(2)}E_X^{(1)*}],$$

$$P_X^{(2)}=2\chi E_Y^{(1)}E_Z^{(1)},$$

$$P_Y^{(2)}=2\chi E_X^{(1)}E_Z^{(1)},$$

$$P_Z^{(2)}=2\chi E_X^{(1)}E_Y^{(1)}.$$

To get the polarization components in the ''light'' system of coordinates, it is sufficient to use the following relations:

$$P_x=P_X\cos\theta\cos\varphi+P_Y\cos\theta\sin\varphi-P_Z\sin\theta,$$

$$P_y=-P_X\sin\varphi+P_Y\cos\varphi,$$

$$E_X=E_x\cos\theta\cos\varphi-E_y\sin\varphi+E_z\sin\theta\cos\varphi=E_xn_e^2\frac{\sin 2\varphi}{2\varepsilon_\perp}-E_y\sin\varphi+\frac{i}{\Gamma}n_e^2\frac{\sin\theta\cos\varphi}{\varepsilon_\perp\varepsilon_\parallel}\times\left(n_e^2\frac{\partial E_x}{\partial x}+\varepsilon_\perp\frac{\partial E_y}{\partial y}\right),$$

$$E_Y=E_x\cos\theta\sin\varphi+E_y\cos\varphi+E_z\sin\theta\sin\varphi=E_xn_e^2\frac{\cos\theta\sin\varphi}{\varepsilon_\perp}+E_y\cos\varphi+\frac{i}{\Gamma}n_e^2\frac{\sin\theta\sin\varphi}{\varepsilon_\perp\varepsilon_\parallel}\times\left(n_e^2\frac{\partial E_x}{\partial x}+\varepsilon_\perp\frac{\partial E_y}{\partial y}\right),$$

$$E_Z=E_z\cos\theta-E_x\sin\theta=-E_xn_e^2\frac{\sin\theta}{\varepsilon_\parallel}+\frac{i}{\Gamma}n_e^2\frac{\cos\theta}{\varepsilon_\perp\varepsilon_\parallel}\left(n_e^2\frac{\partial E_x}{\partial x}+\varepsilon_\perp\frac{\partial E_y}{\partial y}\right).$$

Then, neglecting terms of the second order in the small parameter $(kw)^{-1}$, we can present these components in the form

$$P_x^{(1)}=d_{xxx}^{(1)}E_x^{(2)}E_x^{(1)*}+d_{xyx}^{(1)}E_y^{(2)}E_x^{(1)*}+d_{xxy}^{(1)}E_x^{(2)}E_y^{(1)*}+\frac{i}{\Gamma_1}\left(d_{xxx'}^{(1)}E_x^{(2)}\frac{\partial E_x^{(1)*}}{\partial x}+d_{xxy'}^{(1)}E_x^{(2)}\frac{\partial E_y^{(1)*}}{\partial y}+d_{xyx'}^{(1)}E_y^{(2)}\frac{\partial E_x^{(1)*}}{\partial x}+d_{xyy'}^{(1)}E_y^{(2)}\frac{\partial E_y^{(1)*}}{\partial y}\right)+\frac{i}{\Gamma_2}\left(d_{xx'x}^{(1)}\frac{\partial E_x^{(2)}}{\partial x}E_x^{(1)*}+d_{xy'x}^{(1)}\frac{\partial E_y^{(2)}}{\partial y}E_x^{(1)*}+d_{xx'y}^{(1)}\frac{\partial E_x^{(2)}}{\partial x}E_y^{(1)*}+d_{xy'y}^{(1)}\frac{\partial E_y^{(2)}}{\partial y}E_y^{(1)*}\right), \tag{17}$$

$$P_y^{(1)} = d_{yxx}^{(1)} E_x^{(2)} E_x^{(1)*} + d_{yyx}^{(1)} E_y^{(2)} E_x^{(1)*} + d_{yxy}^{(1)} E_x^{(2)} E_y^{(1)*} + \frac{i}{\Gamma_1}\left(d_{yxx'}^{(1)} E_x^{(2)} \frac{\partial E_x^{(1)*}}{\partial x} + d_{yxy'}^{(1)} E_x^{(2)} \frac{\partial E_y^{(1)*}}{\partial y} + d_{yyx'}^{(1)} E_y^{(2)} \frac{\partial E_x^{(1)*}}{\partial x} + d_{yyy'}^{(1)} E_y^{(2)} \frac{\partial E_y^{(1)*}}{\partial y} \right) + \frac{i}{\Gamma_2}\left(d_{yx'x}^{(1)} \frac{\partial E_x^{(2)}}{\partial x} E_x^{(1)*} + d_{yy'x}^{(1)} \frac{\partial E_y^{(2)}}{\partial y} E_x^{(1)*} + d_{yx'y}^{(1)} \frac{\partial E_x^{(2)}}{\partial x} E_y^{(1)*} + d_{yy'y}^{(1)} \frac{\partial E_y^{(2)}}{\partial y} E_y^{(1)*} \right), \tag{18}$$

$$P_x^{(2)} = d_{xxx}^{(2)} E_x^{(1)2} + d_{xxy}^{(2)} E_x^{(1)} E_y^{(1)} + d_{xyy}^{(2)} E_y^{(1)2} + \frac{i}{\Gamma_1}\left(d_{xxx'}^{(2)} E_x^{(1)} \frac{\partial E_x^{(1)}}{\partial x} + d_{xxy'}^{(2)} E_x^{(1)} \frac{\partial E_y^{(1)}}{\partial y} + d_{xyx'}^{(2)} E_y^{(1)} \frac{\partial E_x^{(1)}}{\partial x} + d_{xyy'}^{(2)} E_y^{(1)} \frac{\partial E_y^{(1)}}{\partial y} \right), \tag{19}$$

$$P_y^{(2)} = d_{yxx}^{(2)} E_x^{(1)2} + d_{yxy}^{(2)} E_x^{(1)} E_y^{(1)} + d_{yyy}^{(2)} E_y^{(1)2} + \frac{i}{\Gamma_1}\left(d_{yxx'}^{(2)} E_x^{(1)} \frac{\partial E_x^{(1)}}{\partial x} + d_{yxy'}^{(2)} E_x^{(1)} \frac{\partial E_y^{(1)}}{\partial y} + d_{yyx'}^{(2)} E_y^{(1)} \frac{\partial E_x^{(1)}}{\partial x} + d_{yyy'}^{(2)} E_y^{(1)} \frac{\partial E_y^{(1)}}{\partial y} \right). \tag{20}$$

It is straightforward to get expressions for the coefficients d. For example,

$$d_{xxx}^{(2)} = -\frac{1}{2}\chi \sin 2\theta \sin 2\varphi \frac{\varepsilon_\parallel[\varepsilon_\perp \cos\theta + (\varepsilon_\perp + \varepsilon_\parallel)\sin\varphi]}{(\varepsilon_\perp \sin^2\theta + \varepsilon_\parallel \cos^2\theta)^2},$$

$$d_{xxy}^{(2)} = 2\chi \sin\theta \frac{\varepsilon_\perp \varepsilon_\parallel[-\varepsilon_\perp \cos\theta \cos 2\varphi + \varepsilon_\parallel(\cos\theta \sin\varphi - \cos^2\varphi)\sin\varphi]}{(\varepsilon_\perp \sin^2\theta + \varepsilon_\parallel \cos^2\theta)^2},$$

$$d_{yyy}^{(2)} = 0, \quad d_{xyy}^{(2)} = \chi \sin 2\varphi \sin\theta, \quad d_{yxy}^{(2)} = \chi \sin\theta \frac{\varepsilon_\perp^2 \varepsilon_\parallel[\sin 2\varphi + 2\cos\varphi]}{(\varepsilon_\perp \sin^2\theta + \varepsilon_\parallel \cos^2\theta)^2}.$$

IV. FIELD POLARIZATION STATE

Equations (15) and (16) represent a system of four coupled equations for the field two frequencies $j=1,2$ and the two transverse components x,y. They are valid for any type of phase matching, and include walk-off effects connected with difference in directions of the light-wave vector and Poynting vector for extraordinary waves (field x components). The equations include lowest-order nonparaxial terms (see a nonlocal form of the medium nonlinear polarization, Eqs. (17)–(20), but they are not the main influence on the soliton characteristics. The most remarkable in Eqs. (15) and (16) is appearance of ''additional'' terms that mix the field x- and y-components. Note that these terms are proportional to the factor of anisotropy $(\varepsilon_\perp^{(j)} - \varepsilon_\parallel^{(j)})$ and are absent under approximation of weak anisotropy [5]. However, this factor is not small for a number of widely used nonlinear crystals, and in this case it is necessary to solve the full system (15) and (16). And even in the case of weak anisotropy, one has to use Eqs. (15) and (16) for determination of the field polarization state that will be the goal of our following consideration.

Let us consider the so-called $oo \to e$ interaction when in the zeroth-order approximation $(\varepsilon_\perp^{(j)} - \varepsilon_\parallel^{(j)} \to 0)$ radiation at the fundamental frequency is an ordinary wave $(E_x^{(1)} = 0, E_y^{(1)} \neq 0)$, and at the second harmonic it is an extraordinary wave $(E_x^{(2)} \neq 0, E_y^{(2)} = 0)$. The cylindrically symmetric components $E_y^{(1)}(\rho)$ and $E_x^{(2)}(\rho)$ $(\rho = \sqrt{x^2+y^2})$, were found numerically for the first time in Ref. [1], and they are given

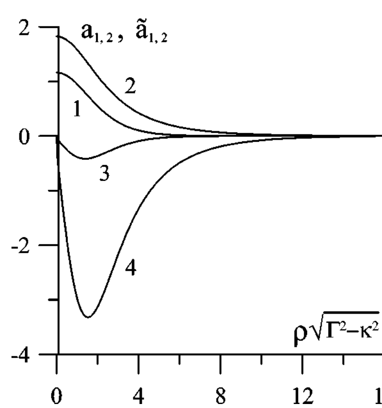

FIG. 1. Radial profiles of dimensionless amplitudes for scalar soliton $a_{1,2}$ (curves 1, 2) and for additional field vectorial components $\tilde{a}_{1,2}$ (curves 3, 4), for relative nonlinear coefficients in Eqs. (24) and (25) we used values $d_{xxx}^{(1)}/d_{yxy}^{(1)} = 0.5$, and $\frac{1}{2}(d_{yxy}^{(2)}/d_{xyy}^{(2)}) \times (\beta_{xy}^{(1)}/\beta_{xy}^{(2)}) = 0.3$.

in Fig. 1. The field additional components can be found by a perturbation approach similar to one used in Ref. [7] from the linear equations

$$\nabla_{\perp}^2 E_y^{(2)} - (\Gamma_2^2 - k_2^2) E_y^{(2)} + d_{yxy}^{(2)} E_y^{(1)} E_x^{(1)} + i\beta_{xy}^{(2)} \frac{\partial E_x^{(2)}}{\partial y} = 0, \tag{21}$$

$$\nabla_{\perp}^2 E_x^{(1)} - (\Gamma_1^2 - k_1^2) E_x^{(1)} + d_{xxx}^{(1)} E_x^{(2)} E_x^{(1)*} + i\beta_{xy}^{(1)} \frac{\partial E_y^{(1)}}{\partial y} = 0, \tag{22}$$

where $k_1 = \omega_1 / c n_{o1}$, $k_2 = \omega_2 / c n_{e2}$. The first-order terms of Eqs. (3) and (7) only were kept here. The right-hand sides of these equations are functions known from the zeroth approximation, and in the cylindrical coordinates (ρ,α) they depend on the angle as $\sin\alpha$. Therefore, it is possible to separate variables

$$E_y^{(2)} = i\beta_{xy}^{(2)} R_y^{(2)}(\rho)\sin\alpha, \quad E_x^{(1)} = i\beta_{xy}^{(1)} R_x^{(1)}(\rho)\sin\alpha, \tag{23}$$

and to get the following equations for the radial functions

$$\frac{d^2 R_y^{(2)}}{d\rho^2} + \frac{1}{\rho}\frac{dR_y^{(2)}}{d\rho} - (\Gamma_2^2 - k_2^2) R_y^{(2)} + (d_{yxy}^{(2)} \beta_{xy}^{(1)} / \beta_{xy}^{(2)}) E_y^{(1)} R_x^{(1)} = -\frac{dE_x^{(2)}}{d\rho}, \tag{24}$$

$$\frac{d^2 R_x^{(1)}}{d\rho^2} + \frac{1}{\rho}\frac{dR_x^{(1)}}{d\rho} - (\Gamma_1^2 - k_1^2) R_x^{(1)} + d_{xxx}^{(1)} E_x^{(2)} R_x^{(1)*} = -\frac{dE_y^{(1)}}{d\rho}. \tag{25}$$

Boundary conditions to these ordinary differential equations are natural (the radial functions decay at $\rho \to \infty$ and are finite at $\rho \to 0$). Note that according to Eqs. (23) the ''additional'' components are shifted in phase at $\pi/2$ as compared with the main components. Results of numerical solutions of Eqs. (24) and (25) are presented in Fig. 1. Here we present dimensionless amplitudes a_1, a_2 for soliton with parameters $(\Gamma_2^2 - k_2^2)/(\Gamma_1^2 - k_1^2) = 0.15$ and dimensionless amplitudes $\tilde{a}_1$, $\tilde{a}_2$ for field additional vectorial components depending on dimensionless coordinate

$$a_1 = \frac{E_y^{(1)} \sqrt{d_{yxy}^{(1)} d_{xyy}^{(2)}}}{(\Gamma_1^2 - k_1^2)}, \quad a_2 = \frac{E_x^{(2)} d_{yxy}^{(1)}}{(\Gamma_1^2 - k_1^2)},$$

$$\tilde{a}_1 = \frac{R_x^{(1)}}{\beta_{xy}^{(1)}} \sqrt{\frac{d_{yxy}^{(1)} d_{xyy}^{(2)}}{(\Gamma_1^2 - k_1^2)}}, \quad \tilde{a}_2 = \frac{R_y^{(2)} d_{yxy}^{(1)}}{4\beta_{xy}^{(2)} \sqrt{\Gamma_1^2 - k_1^2}}.$$

Note that now all the components of the fields at both fundamental and the second harmonics are nonzero. Therefore, polarization state at both frequencies is not linearly, but elliptically polarized, and polarization state changes over the soliton transverse section. Only at the axis $(\rho = 0)$ the field polarization is linear. Polarization ellipse is situated approximately in the plane (y,z) for the fundamental wave and in the plane (x,z) for the second harmonics, but its goes off these planes with account of the small field additional components.

The ratio of the ''additional'' electric field component $R_x^{(1)}$ to the main component $E_y^{(1)}$ is proportional to the ratio $\tilde{a}_1 / a_1$,

$$\frac{R_x^{(1)}}{E_y^{(1)}} = \frac{\tilde{a}_1}{a_1} \frac{n_o^2/n_e^2 - 1}{\sqrt{1 - k_1^2/\Gamma_1^2}} \frac{\sin^3\theta\cos\theta}{1 + (n_o^2/n_e^2 - 1)\sin^4\theta}.$$

Let us present estimations for the case of $oo \to e$ interaction in uniaxial KDP crystal. For the fundamental harmonic we have $\lambda = 1.06\ \mu$m, $n_o = 1.4939$, $n_e = 1.4599$ phase-matching angle $\theta \cong 41°$, coefficient of nonlinearity is given by relation $k_1^{-2} |d_{yxy}^{(1)} E_x^{(2)}| \sim 0.13\sqrt{S[\mathrm{MW/cm}^2]}$, where S is the Poynting vector, or the radiation intensity. The value $1 - k^2/\Gamma^2 \sim k^{-2} |d_{yxy}^{(1)} E_x^{(2)}| \sim 1/(kw)^2$ is a measure of the soliton nonparaxiality. The maximum ratio $|\tilde{a}_1 / a_1| = 0.36$ (see Fig. 1), therefore $|R_x^{(1)}/E_y^{(1)}| \cong (10^{-2}/S[\mathrm{W/cm}^2])^{1/4}$. It is naturally to introduce the critical intensity $S_{cr} = 10^{-2}[\mathrm{W/cm}^2]$, which corresponds to the soliton width $kw \sim 2800$. Then for solitons with width in the range $1 \lesssim kw \lesssim 2800$ effect of anisotropy (appearance of essential additional field components) becomes important.

V. CONCLUSIONS

We have derived the envelope equations that describe the full polarization structure of stationary spatial 2D quadratic optical solitons. These equations include a number of terms additional to the standard ones and of the same order of magnitude as terms describing so-called walk-off effects. In the case of weak anisotropy the governing equations allow a simple solution by the perturbative approach that reveals nontrivial state of the soliton polarization state. For experimental verification of the results presented, it is simpler to register appearance and angular dependence of polarization components of quadratic solitons excited by radiation with linear state of polarization for the fundamental frequency and its second harmonics.

The governing equations admit generalization with a standard taking into account propagation effects. Note, however, that in the case of a weak anisotropy we do not need soliton stability analysis. In fact, we deal in the zeroth order with a stable soliton, so its stability cannot be changed as a result of small perturbations. Other straightforward generalization is the case of optical parametric oscillators that are very important for different applications [2].

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